Unit Roots and Identification in Autoregressive Panel Data Models: A Comparison of Alternative Tests

Stephen Bond
Institute for Fiscal Studies and Nuffield College, Oxford

Céline Nauges
LEERNA-INRA Toulouse and University College London

Frank Windmeijer
Institute for Fiscal Studies, 7 Ridgmount Street, London WC1E 7AE

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Abstract

We compare the finite sample behaviour of various unit root tests for micro panels where the number of individuals is typically large, but the number of time periods is often very small. As in this case some econometric estimators do not identify the parameters of interest when the processes are random walks, it is important to test for unit roots/identification. We find that a t-test based on OLS estimation results provides a simple robust test with high power for cases when the variance of the unobserved heterogeneity is relatively small. Its behaviour is similar to the underidentification test as proposed by Arellano, Hansen and Sentana (1999) for the GMM estimator on a first-differenced model.

JEL Classification: C12, C23

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1 Introduction

The time series properties of economic series (orders of integration and cointegration) are often of considerable interest. In micro panels, where the number of individuals is typically large but the number of time periods is often very small, these properties can also be crucial for identification of econometric models. Where differencing transformations are used to eliminate unobserved individual-specific effects, identification requires the existence of instrumental variables that are correlated with first-differences of the series. In the extreme case of a pure random walk, lagged values of the series are uncorrelated with first-differences, and the widely used first-differenced instrumental variables estimators will provide no information on the parameter of interest.

It is therefore important to assess the time series properties of the series under consideration. Various test procedures have been proposed in the literature recently. In this paper we will develop these tests, paying particular attention to those that are appropriate for micro panels, and relating unit root tests to the issue of identification.

To focus our discussion, we will concentrate on testing the value of α in the first-order autoregressive model with unobserved individual-specific effects η_i and serially uncorrelated disturbances v_{it}

$$y_{it} = \alpha y_{it-1} + (1-\alpha)\,\eta_i + v_{it},\ |\alpha| \leq 1$$

for $i = 1, ..., N$ and $t = 2, ..., T$. This model reduces to a random walk when $\alpha = 1$.

Properties of estimators of α in this setting are well known. For $\alpha < 1$, ordinary least squares gives an upward biased and inconsistent estimate, due to the correlation between the lagged dependent variable and the omitted individual-specific effect. However for $\alpha = 1$, ordinary least squares is consistent. Regardless of the true value of α, within groups (or least squares dummy variables) gives a downward biased estimate, which is inconsistent as N becomes large for fixed T (Nickell, 1981).

A widely used estimator in this context eliminates the individual-specific effects by first-differencing or related transformations, and then uses suitably lagged levels of the series as instruments for the equations in first differences. In the AR(1) example, lagged levels of the dependent variable dated $t-2$ and earlier are available as instruments. These orthogonality conditions can be exploited efficiently in a Generalised Method of Moments (GMM) estimator, which was developed in Holtz-Eakin, Newey and Rosen (1988), and Arellano and Bond (1991).

However for the parameter to be identified using this first-differenced GMM estimator it is important that the instruments are correlated with the endogenous variable in the first-differenced equations. More generally, if the instruments are weak in the sense of Staiger and Stock (1997), simulation studies have shown that the finite sample properties of the first-differenced GMM estimator can be very poor. This happens when the series are highly persistent. In the extreme case when the series are a random walk

$$y_{it} = y_{it-1} + v_{it}$$

there is no correlation between $\Delta y_{it-1} = v_{it-1}$ and lagged levels of the series dated $t-2$ and earlier. In this case the first-differenced GMM estimator does not identify α, and will not provide any information about this parameter even in very large samples.

Blundell and Bond (1998) proposed an alternative GMM estimator that imposes a restriction on the initial conditions (y_{i1}). They only considered stationary models with $\alpha < 1$. However, following Binder, Hsiao and Pesaran (2000) we can extend this to include the unit root case. The required properties of the initial conditions are

$$\begin{aligned} E\left[(y_{i1} - \eta_i)\,\eta_i\right] &= 0 \quad \text{if } \alpha < 1 \\ Var\,(y_{i1}) &< \infty \quad \text{if } \alpha = 1. \end{aligned}$$

The former restriction ensures that Δy_{it} is uncorrelated with η_i for stationary processes, and the latter restriction ensures that y_{it} is correlated with Δy_{it-1} for non-stationary processes. Then regardless of its true value, the parameter α is identified using lagged differences Δy_{is} dated $t-1$ and earlier as instrumental variables for the levels equations. For $\alpha < 1$ there is additional information in the first-differenced equations, and both sets of moment conditions are combined in the 'system' GMM estimator developed in Blundell and Bond (1998).

Testing for a unit root in micro panels is thus important in order to assess whether the first-differenced GMM estimator is identified or whether other estimators need to be considered. The time series properties of particular series may be of independent interest, and when applied to residuals from regression models, these procedures will also form the basis for cointegration tests. Most of the recent literature on testing for unit roots in panel data has focused on panels with a small number of cross-sectional units, N, observed over many time periods, T, typical of panel data for countries or industries.[1] A much smaller literature exists on testing for unit roots when the cross-sectional dimension of the panel is large, but the time dimension is small and treated as fixed, the type of panel data often encountered when individuals, households or firms are surveyed. We will focus on the latter case.

[1] For a recent survey, see Baltagi and Kao (2000).

These tests can be divided into two basic types: those based on estimates of α that are consistent both under the unit root null and under the alternative; and those based on estimators which are only consistent (or have biases that can be characterised) under the null.

As the OLS estimator is consistent under the null of a random walk, a simple test is a t-test on the estimated OLS parameter. The asymptotic distribution of this t-test is standard in the case of micro panels, where we rely on $N \to \infty$ with T fixed to derive asymptotic approximations. The power of this test will however be influenced by the variance of the unobserved heterogeneity in the model under the alternative that $\alpha < 1$. As the OLS estimator is biased upward under the alternative, the power to reject $\alpha = 1$ will be low when $Var(\eta_i)/Var(v_{it})$ is large. An alternative to OLS is to use a two-step GMM estimator, using lagged levels y_{is} dated $t-1$ and earlier as instruments for the levels equations. This will be more efficient than OLS in the presence of heteroscedasticity, and can be expected to increase the power of the t-test in this case. The GMM test for overidentifying restrictions also provides additional information on the appropriateness of the random walk model specification.

Motivated by concern over the power of the simple OLS test when $Var(\eta_i)/Var(v_{it})$ is high, Breitung and Meyer (1994) proposed an alternative test based on the OLS estimator in a long-differenced model. In this model the OLS estimator has an asymptotic bias that is independent of the variance of the individual effects, and can be calculated under specific assumptions. Similarly Harris and Tzavalis (1999) derived the asymptotic bias and variance of the within groups estimator for the random walk model, and proposed a test for the null of a unit root based on the bias corrected estimates. These corrections are derived for a model with normal, homoscedastic errors, and the tests may not be robust to deviations from these assumptions. In many applications it is likely that there is conditional heteroscedasticity of complex forms that are difficult to characterise. The simpler tests based on OLS or GMM estimates in the untransformed model will be asymptotically robust to this. Moreover there is likely to be a power trade-off between tests based on estimators whose bias under the alternative does not depend on the unobserved heterogeneity, but which are less efficient under the null.

A related test is developed by Arellano, Hansen and Sentana (1999). They propose a general test for the identification of the parameters in models estimated by GMM. Applied to the first-differenced GMM estimator for the AR(1) model in equation (1), this becomes a test for unit roots. Whilst their test will be useful in a much wider range of models, it will be interesting to compare its performance in this setting, where a range of alternative tests of the same hypothesis are available.

We find that a t-test based on OLS estimation results provides a simple robust test with high power for cases when the variance of the unobserved heterogeneity is relatively small. Its behaviour is similar to the underidentification test as proposed by Arellano, Hansen and Sentana (1999) for the first-differenced GMM estimator. The test proposed by Breitung and Meyer (1994) does not lose power with increasing variance, but its power can be quite low. The test by Harris and Tzavalis (1999) is shown to be very sensitive to features of the underlying data. Especially when there is heteroscedasticity over time, its size properties are severely distorted.

The next section introduces the model and GMM estimators. Section 3 describes the underidentification tests and Section 4 the alternative unit root testing procedures. Section 5 presents some Monte Carlo results and Section 6 concludes.

2 Model and GMM Estimation

Consider the simple dynamic AR(1) panel data model

$$\begin{aligned} y_{it} &= \alpha y_{it-1} + u_{it} \\ u_{it} &= (1-\alpha)\,\eta_i + v_{it}, \end{aligned} \tag{1}$$

for $i = 1, ..., N$ and $t = 2, ..., T$; N is large and T is fixed. Note that there are no individual effects when $\alpha = 1$. The observations are independent across individuals and the error term satisfies

$$E(\eta_i) = 0,\ E(v_{it}) = 0 \quad \text{for } i = 1, ..., N \text{ and } t = 2..., T$$

and

$$E(v_{it} v_{is}) = 0 \quad \text{for } i = 1, ..., N \text{ and } t \neq s.$$

If it is only assumed that the v_{it} are uncorrelated with y_{i1}:

$$E(y_{i1} v_{it}) = 0 \quad \text{for } i = 1, ..., N \text{ and } t = 2..., T,$$

then there are the following $(T-1)(T-2)/2$ linear moment conditions available for the estimation of α by GMM

$$E(y_{is} \Delta u_{it}) = 0 \quad \text{for } t = 3, \ldots T,\ s = 1, \ldots, t-2, \tag{2}$$

where $\Delta u_{it} = u_{it} - u_{it-1} = \Delta y_{it} - \alpha \Delta y_{it-1}$, see for example Arellano-Bond (1991). Specifying the instrument set as

$$Z_i^D = \begin{bmatrix} y_{i1} & 0 & 0 & \ldots & 0 & \ldots & 0 \\ 0 & y_{i1} & y_{i2} & \ldots & 0 & \ldots & 0 \\ 0 & 0 & 0 & \ddots & 0 & \ldots & 0 \\ 0 & 0 & 0 & \ldots & y_{i1} & \ldots & y_{i(T-2)} \end{bmatrix},$$

the GMM estimator minimises

$$\left(\frac{1}{N}\sum_{i=1}^{N} Z_i^{D\prime} \Delta u_i\right)' W_N \left(\frac{1}{N}\sum_{i=1}^{N} Z_i^{D\prime} \Delta u_i\right)$$

where $\Delta u_i = [\Delta u_{i3}, \Delta u_{i4}, \ldots, \Delta u_{iT}]'$, and W_N is a positive semi-definite weight matrix that converges to a positive definite matrix W as $N \to \infty$. Under general conditions, an optimal two-step estimator is based on the weight matrix

$$W_N = \left(\frac{1}{N} \sum_{i=1}^{N} Z_i^{D\prime} \widehat{\Delta u_i} \widehat{\Delta u}_i' Z_i^D \right)^{-1},$$

where $\widehat{\Delta u_i}$ are the residuals based on an initial consistent estimator for α.

If further an error components structure on the error term and mean stationarity on the process are imposed, implying that

$$E(\eta_i v_{it}) = 0 \quad \text{for } i = 1, ..., N \text{ and } t = 2..., T$$

$$y_{i1} = \eta_i + \varepsilon_i \quad \text{for } i = 1, ..., N$$

and

$$E(\varepsilon_i) = E(\eta_i \varepsilon_i) = 0 \quad \text{for } i = 1, ..., N,$$

see for example Ahn-Schmidt (1995), Arellano-Bover (1995) and Blundell-Bond (1998), there are the following extra $(T-2)$ linear moment conditions available:

$$E(u_{it} \Delta y_{it-1}) = 0 \quad \text{for } t = 3, ..., T. \tag{3}$$

The so-called system GMM estimator for α is obtained by stacking the residuals from the differenced and level equations, and extending the instrument matrix to

$$Z_i^S = \begin{bmatrix} Z_i^D & 0 & \cdots & 0 \\ 0 & \Delta y_{i2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Delta y_{i(T-1)} \end{bmatrix}.$$

The system GMM estimator combines the moment conditions for the model in first-differences with those for the model in levels. A simpler (and less efficient) GMM levels estimator, that is based on the $(T-1)(T-2)/2$ moment conditions

$$E\left(u_{it}\Delta y_{i,t-s}\right)=0;\ \text{for } t=3,...,T \text{ and } 1\leq s\leq t-2, \tag{4}$$

relates only to the equations in levels. These can be expressed as

$$E\left(Z_i^{L\prime}u_i\right)=0,$$

where Z_i^L is the $(T-2)\times(T-1)(T-2)/2$ matrix given by

$$Z_i^L=\begin{bmatrix} \Delta y_{i2} & 0 & 0 & ... & 0 & ... & 0 \\ 0 & \Delta y_{i2} & \Delta y_{i3} & ... & 0 & ... & 0 \\ . & . & . & ... & . & ... & . \\ 0 & 0 & 0 & ... & \Delta y_{i2} & ... & \Delta y_{iT-1} \end{bmatrix},$$

and u_i is the $(T-2)$ vector $(u_{i3},u_{i4},...,u_{iT})'$.

3 Unit Roots and Identification

3.1 First-Differenced GMM

For the first-differenced GMM estimator that utilises moment conditions (2), the endogenous lagged differences $\Delta y_{i,t-1}$ are instrumented by lagged levels $y_{i1},...,y_{it-2}$. Clearly, when $\alpha=1$, the rank condition is not satisfied as the instruments are uncorrelated with the endogenous variable, and therefore α is not identified in this case. Arellano, Hansen and Sentana (1999), henceforth AHS, propose a general test of the identification of the parameters in models estimated by GMM. For the simple AR(1) panel data model their test for underidentification is a test for the validity of the moment conditions

$$E\left(y_i^{t-1}\Delta y_{it}\right)=0, \tag{5}$$

where $y_i^{t-1}=(y_{i1},...,y_{it-1})'$. The Sargan test statistic for overidentifying restrictions has an asymptotic χ^2 distribution with $T(T-1)/2$ degrees of freedom when the model is underidentified and the moment conditions (5) are valid. When the Sargan test rejects, the model is not underidentified. For this model it is clear that a test for identification is equivalent to a test for a unit root, $H_0 : \alpha = 1$, and we will compare the performance of the AHS test for underidentification to various tests for a unit root, as described in the next section. The AHS test is equivalent to the Anderson-Rubin test for $H_0 : \alpha = 1$ in the first differenced GMM model.

3.2 System GMM

For the system estimator the $T-2$ extra moment conditions (3) remain valid when $\alpha = 1$,[2] even though the process is clearly not mean-stationary in this case. Consider the first stage regression for the levels equation, when $T = 3$,

$$y_{i2} = \pi \Delta y_{i2} + r_i.$$

If $\alpha = 1$, it follows that $\pi = 1$ and $r_i = y_{i1}$. Denote TP the number of periods the process has been in existence before the sample is drawn. Then, for any fixed TP, $plim_{N\to\infty}\widehat{\pi}_{OLS} = 1$, and the model is (asymptotically) identified. If TP goes to infinity at a faster rate than N, then $plim_{N\to\infty}\widehat{\pi}_{OLS}$ does not exist, and the model is not identified, although $\widehat{\pi}_{OLS}$ is centered around 1. For any given sample, the ratio of N to TP determines how well the distribution of the level or system GMM estimator is approximated by its asymptotic distribution. Even if the model is not identified, the level and system estimator will be centered around 1, as the 2SLS estimator will be centered around the mean of the OLS estimator in the levels equation.

The AHS test for underidentification for the GMM levels model using

[2] This is no longer true if there are individual specific drifts.

moment conditions (4) is a test for the moment conditions

$$E\left(y_{it}\Delta y_i^t\right) = 0,$$

where $\Delta y_i^t = (\Delta y_{i2}, ..., \Delta y_{it})'$.

4 Tests for Unit Roots

4.1 OLS

Under the null $H_0 : \alpha = 1$, the OLS estimator in model (1) is unbiased and consistent, and a simple t-test based on OLS results is given by

$$t_{OLS} = \frac{\widehat{\alpha}_{OLS} - 1}{\sqrt{Var\left(\widehat{\alpha}_{OLS}\right)}}$$

where

$$Var\left(\widehat{\alpha}_{OLS}\right) = \left(y'_{-1}y_{-1}\right)^{-1} \sum_{i=1}^{N} y'_{i,-1}e_i e'_i y_{i,-1} \left(y'_{-1}y_{-1}\right)^{-1},$$

with $e_i = y_i - y_{i,-1}\widehat{\alpha}_{OLS}$, $y_i = (y_{i2}, ..., y_{iT})'$, $y_{i,-1} = (y_{i1}, ..., y_{iT-1})'$, and $y_{-1} = (y'_1, y'_2, ..., y'_N)'$. Under the null, t_{OLS} has an asymptotic standard normal distribution. Under the alternative, $\alpha < 1$, the OLS estimator is biased upwards, more so when the variance of η_i is large, and the power of the test will therefore depend on the magnitude of σ_η^2.

4.2 Breitung and Meyer

In response to this sensitivity to σ_η^2 of the simple test based on the OLS estimator in the levels equation, Breitung and Meyer (1994) propose a modified Dickey-Fuller statistic, based on the OLS estimator for α in the transformed model

$$y_{it} - y_{i1} = \alpha\left(y_{it-1} - y_{i1}\right) + \varepsilon_{it}, \quad t = 3, ..., T, \tag{6}$$

where $\varepsilon_{it} = u_{it} - (1-\alpha)(y_{i1} - \eta_i)$. Clearly, the OLS estimator in this model is unbiased when $\alpha = 1$ and the simple t-test is valid under the null of a unit root. When $\alpha < 1$ the OLS estimator is upwards biased. The asymptotic bias is given by:

$$plim_{N\to\infty}\widehat{\alpha}_{BM} = \frac{\alpha+1}{2},$$

which is not a function of η_i. Therefore, the power of the test is not affected by the magnitude of σ^2_η.

4.3 Harris and Tzavalis

Harris and Tzavalis (1999) base a test for the unit root hypothesis on a bias correction of the within groups estimator under the null. Under the assumptions that $v_{it} \sim iid\, N(0,\sigma^2_v)$ and the y_{i1} are fixed observable constants, which implies that y_{i1} is uncorrelated with the sequence $\{v_{it}\}$, Harris and Tzavalis (1999) show that, under the null of a unit root in model (1),

$$\sqrt{N}\left(\widehat{\alpha}_{WG} - 1 - B\right) \to N\left(0, C\right),$$

where $\widehat{\alpha}_{WG}$ is the within groups estimator of α, and B and C are given by[3]

$$\begin{aligned} B &= -\frac{3}{T}; \\ C &= \frac{3\left(17\left(T-1\right)^2 - 20\left(T-1\right) + 17\right)}{5T^3\left(T-2\right)}. \end{aligned}$$

A simple test then is $\left(\widehat{\alpha}_{WG} - 1 - B\right)/\sqrt{C/N}$, which has an asymptotic standard normal distribution under the null.

As the bias correction and derived variance are valid only under homoscedasticity, it is likely that the test performance will be poor under certain forms of heteroscedasticity.

[3] Note that these expressions differ from those in Harris and Tzavalis (1999, p.207) due to the fact that our first observation is y_{i1}, not y_{i0}. Therefore, their panel length T is replaced by $T-1$ in our case.

5 Monte Carlo Results

In this section we present the results of an extensive Monte Carlo study, investigating the properties of the various estimators and test statistics as described in the previous sections.

The general data generating process is

$$y_{it} = \alpha y_{it-1} + (1-\alpha)\,\eta_i + v_{it},$$

with $\eta_i \sim N\left(0, \sigma_\eta^2\right)$, and, initially, $v_{it} \sim N(0,1)$.

1. To investigate the size behaviour of the various tests, and the behaviour of the various estimators, we consider unit root processes for which $y_{i0} = 0$, i.e. there is no pre-sample history, and processes that started up 50 periods ago.

2. To both investigate the behaviour of the estimators under the alternative, and to determine the power of the various test statistics, we consider processes with $\alpha < 1$ with (covariance) stationary initial conditions, $y_{i0} \sim N\left(\eta_i, \frac{\sigma_v^2}{1-\alpha^2}\right)$. We also study the case when the v_{it} are heteroscedastic. In all cases, $N = 200$, $T = 6$.

5.1 Unit Root

Table 1 presents the estimation results for the various estimators when the process is a random walk, for the case of no pre-sample history, the column is labeled $y_0 = 0$, and for the case of 50 pre-sample periods, the column is labeled $y_{-50} = 0$. The estimators considered are the simple OLS estimator in the levels equation (1), denoted OLS in the table; the OLS estimator in the transformed model (6) as proposed by Breitung and Meyer (1994), denoted BM; the simple within groups estimator; and the two-step GMM DIF and

SYS estimators. As expected, the within groups estimator is downward biased. As the correction factor derived by Harris and Tzavalis (1999) is equal to 0.5 in this case, the bias corrected estimator is virtually unbiased. The GMM DIF estimator is very poorly behaved, as this estimator is not identified. The OLS, BM and SYS estimator are virtually unbiased, as expected.

Table 1. Estimation Results, $\alpha = 1$

		$y_0 = 0$	$y_{-50} = 0$
OLS	Mean	0.9995	0.9999
	St Dev	0.0185	0.0044
BM	Mean	0.9993	0.9983
	St Dev	0.0227	0.0232
Within-Groups	Mean	0.4996	0.4982
	St Dev	0.0349	0.0351
GMM DIF	Mean	0.1853	0.1970
	St Dev	0.4275	0.4336
GMM SYS	Mean	1.0002	0.9940
	St Dev	0.0243	0.0341

Notes: based on 5,000 replications. $N = 200$, $T = 6$.

Table 2 presents the size properties for the various tests for unit roots and the overidentification test of Arellano, Hansen and Sentana (1999). For the two-step GMM estimators the estimated variances have been corrected using the small sample adjustment as developed in Windmeijer (2000).

The OLS, BM and HT tests have the correct size of 0.05 for both processes. The t-test based on the GMM DIF estimation results has of course very poor size properties. When $y_0 = 0$ the GMM SYS t-test is slightly undersized, which gets worse when there are 50 pre-sample periods, as the parameter gets less well identified. The underidentification test statistic, denoted UI - DIF and UI - LEV, clearly shows that the DIF estimator is not identified. It also confirms that the LEV (and therefore SYS) estimator is identified when $y_0 = 0$ whereas it is less so when there are 50 pre-sample periods.

Table 2. Test Results, $H_0 : \alpha = 1$
$H_1 : \alpha < 1$, size $= 0.05$

	$y_0 = 0$	$y_{-50} = 0$
OLS	0.0542	0.0588
BM	0.0552	0.0652
HT	0.0550	0.0560
GMM DIF	0.5468	0.5388
GMM SYS	0.0406	0.0298
UI - DIF	0.0568	0.0572
UI - LEV	1.0000	0.3834

Notes: based on 5,000 replications, $N = 200$, $T = 6$.
All tests are one-sided tests at the 5% level.
All tests based on GMM estimates use two-step estimates and corrected two-step standard errors.
In UI tests, the null is underidentification.

5.2 Stationarity, $\alpha < 1$

Tables 3 and 4 present estimation and test results respectively for the alternative of a covariance stationary process. Values for α considered are 0.90, 0.95 and 0.98. The two values of σ_η^2 that are considered are 1 and 100 respectively. It is clear that the OLS and BM estimators are upward biased, whereas the within groups and GMM DIF estimators are downward biased. When $\sigma_\eta^2 = 1$ the upward bias of the OLS estimator is small. The GMM SYS estimator displays a small bias, which is negative when $\sigma_\eta^2 = 1$.

Table 3. Estimation Results, covariance stationary initial conditions

		$\alpha = 0.90$		$\alpha = 0.95$		$\alpha = 0.98$	
		$\sigma_\eta = 1$	$\sigma_\eta = 10$	$\sigma_\eta = 1$	$\sigma_\eta = 10$	$\sigma_\eta = 1$	$\sigma_\eta = 10$
OLS	Mean	0.9155	0.9950	0.9541	0.9952	0.9807	0.9959
	St Dev	0.0127	0.0029	0.0095	0.0029	0.0062	0.0028
BM	Mean	0.9487	0.9491	0.9735	0.9742	0.9890	0.9896
	St Dev	0.0238	0.0233	0.0231	0.0231	0.0227	0.0226
Within-Groups	Mean	0.4356	0.4357	0.4666	0.4677	0.4864	0.4871
	St Dev	0.0346	0.0343	0.0352	0.0343	0.0341	0.0342
GMM DIF	Mean	0.8504	0.5013	0.8496	0.4338	0.7382	0.3862
	St Dev	0.1213	0.3496	0.1655	0.4041	0.2822	0.4226
GMM SYS	Mean	0.8865	0.9270	0.9313	0.9513	0.9557	0.9684
	St Dev	0.0555	0.0677	0.0586	0.0672	0.0657	0.0641

Notes: based on 5,000 replications. $N = 200$, $T = 6$.

Table 4 shows the rejection frequencies for the tests of a unit root. As expected, the simple OLS t-test performs very well when $\sigma_\eta = 1$. Its power is considerably smaller when $\sigma_\eta = 10$. It is clear that the power of the BM and HT tests are not affected by increases in σ_η. However, the power of these tests are quite low for higher values of α, and the value of σ_η has to be very large for the standard OLS t-test to have lower power.[4] The GMM SYS t-test also has very low power at high values of α. The UI DIF finds that the model

[4]For example, for $\alpha = 0.95$ and $\sigma_\eta = 20$ the rejection frequency for the OLS one-sided t-test is 0.2086, for $\alpha = 0.98$ and $\sigma_\eta = 50$ it is 0.0968.

is identified less frequently than the OLS t-test rejects a unit root, especially when $\sigma_\eta = 10$. Also in that case, the underidentification test for the levels moment conditions almost never rejects the null of underidentification.

Table 4. Test Results, stationary model, $H_0 : \alpha = 1$, $H_1 : \alpha < 1$

	$\alpha = 0.90$		$\alpha = 0.95$		$\alpha = 0.98$	
	$\sigma_\eta = 1$	$\sigma_\eta = 10$	$\sigma_\eta = 1$	$\sigma_\eta = 10$	$\sigma_\eta = 1$	$\sigma_\eta = 10$
OLS	1.0000	0.5510	0.9992	0.4926	0.9328	0.4358
BM	0.7110	0.7060	0.3136	0.3040	0.1224	0.1166
HT	0.5878	0.5784	0.2522	0.2408	0.1040	0.0984
GMM DIF	0.3668	0.4318	0.2614	0.4258	0.2498	0.4256
GMM SYS	0.6958	0.2778	0.2758	0.1198	0.1150	0.0736
UI - DIF	0.9984	0.3312	0.9014	0.1716	0.4232	0.1128
UI - LEV	0.9174	0.0630	0.5372	0.0288	0.1324	0.0206

Notes: based on 5,000 replications. $N = 200$, $T = 6$.
All tests are one-sided tests at the 5% level.
All tests based on GMM estimates use two-step estimates and corrected two-step standard errors.
In UI tests, the null is underidentification.

5.3 Heteroscedasticity

We consider two separate cases, heteroscedasticity over time and heteroscedasticity over individuals. The DGP for the heteroscedasticity over time case is given by

$$\begin{aligned} y_{it} &= \alpha y_{it-1} + (1-\alpha)\eta_i + v_{it} \\ v_{it} &= 3\tau_t \varepsilon_{it} \end{aligned}$$

$$
\begin{aligned}
\tau_t &\sim U[0,1] \\
\varepsilon_{it} &\sim N(0,1),
\end{aligned}
$$

and by

$$
\begin{aligned}
v_{it} &= 3\xi_i \varepsilon_{it} \\
\xi_i &\sim U[0,1] \\
\varepsilon_{it} &\sim N(0,1)
\end{aligned}
$$

for the heteroscedasticity over individuals.

Tables 5 and 6 present estimation and test results for the unit root process with heteroscedastic errors, whereas Tables 7 and 8 present the results for stationary processes.

It is clear that the presence of heteroscedasticity does not affect the estimation and most of the test results when there is a unit root. The only test for which the size is now very poor when there is heteroscedasticity over time is the HT test.

Table 5. Estimation Results, $\alpha = 1$, heteroscedastic errors

Het		over time	over ind.
OLS	Mean	0.9988	0.9988
	St Dev	0.0194	0.0196
BM	Mean	0.9987	0.9988
	St Dev	0.0250	0.0228
Within-Groups	Mean	0.4897	0.4988
	St Dev	0.1321	0.0377
GMM DIF	Mean	0.2576	0.2093
	St Dev	0.4266	0.4344
GMM SYS	Mean	0.9989	0.9987
	St Dev	0.0173	0.0240

Notes: based on 5,000 replications. $N = 200$, $T = 6$.

Table 6. Test Results, $\alpha = 1$, heteroscedastic errors

Het	over time	over ind.
OLS	0.0522	0.0612
BM	0.0546	0.0578
HT	0.3204	0.0726
GMM DIF	0.6404	0.5478
GMM SYS	0.0404	0.0422
UI - DIF	0.0618	0.0446
UI - LEV	0.9986	1.0000

Notes: based on 5,000 replications. $N = 200$, $T = 6$.

When the process is stationary, with $\alpha = 0.90$, the power of most test statistics is the same as with homoscedastic errors. The power of the BM test has increased substantially, as the BM estimator has a smaller upward bias in these cases.

Table 7. Estimation Results, $\alpha = 0.90$, heteroscedastic errors

het		over time		over individuals	
		$\sigma_\eta = 1$	$\sigma_\eta = 10$	$\sigma_\eta = 1$	$\sigma_\eta = 10$
OLS	Mean	0.9103	0.9920	0.9093	0.9919
	St Dev	0.0171	0.0051	0.0179	0.0047
BM	Mean	0.9234	0.9241	0.9167	0.9170
	St Dev	0.0323	0.0315	0.0247	0.0250
Within-Groups	Mean	0.4018	0.4035	0.4019	0.4026
	St Dev	0.1352	0.1333	0.0376	0.0381
GMM DIF	Mean	0.8216	0.6721	0.7872	0.6018
	St Dev	0.1570	0.2945	0.1743	0.2892
GMM SYS	Mean	0.8986	0.9024	0.8955	0.8923
	St Dev	0.0203	0.0347	0.0337	0.0508

Notes: based on 5,000 replications. $N = 200$, $T = 6$.

Table 8. Test Results, $\alpha = 0.90$ heteroscedastic errors, $H_0 : \alpha = 1$, $H_1 : \alpha < 1$, size 0.05

het	over time		over ind.	
	$\sigma_\eta = 1$	$\sigma_\eta = 10$	$\sigma_\eta = 1$	$\sigma_\eta = 10$
OLS	0.9990	0.5446	1.0000	0.5378
BM	0.8782	0.8730	0.9674	0.9688
HT	0.5798	0.5826	0.8638	0.8608
GMM DIF	0.4464	0.4564	0.3558	0.4126
GMM SYS	0.9806	0.9264	0.9514	0.8356
UI - DIF	0.9922	0.8990	0.9762	0.5992
UI - LEV	0.9980	0.8402	1.0000	0.5540

Notes: based on 5,000 replications. $N = 200$, $T = 6$.

6 Conclusions

We have compared the finite sample behaviour of various unit root tests for micro panels where the number of individuals is typically large, but the number of time periods is often very small. As in this case some econometric estimators do not identify the parameters of interest when the processes are random walks, it is important to test for unit roots/identification. We find that a t-test based on OLS estimation results provides a simple robust test with high power for cases when the variance of the unobserved heterogeneity is relatively small.[5] Its behaviour is similar to the underidentification test as proposed by Arellano, Hansen and Sentana (1999) for the GMM-DIF model, which is equivalent to a unit root test using the Anderson-Rubin statistic. The test proposed by Breitung and Meyer (1994) does not lose power with

[5] See also Hall and Mairesse (2001).

increasing variance, but its power can be quite low. The test by Harris and Tzavalis (1999) is shown to be very sensitive to the underlying data features. Especially when there is heteroscedasticity over time, its size properties are severly distorted.

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